OUT-OF-EQUILIBRIUM CORRELATIONS AND ENTROPY PRODUCTION IN A DRIVEN GRANULAR FLUID

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CONGRATULATIONS TO ANDREA PUGLISI

CORRELATIONS IN A 2D GRANULAR FLUID ?

GG, A.Sarracino,D.Villamaina, A.Puglisi, *EPL,* 96, (2011)
A.Puglisi, A.Gnoli, GG, A.Sarracino,D.Villamaina, *J. Chem. Phys.* 136, (2012)

ENERGY GAIN :
vibrating vessel

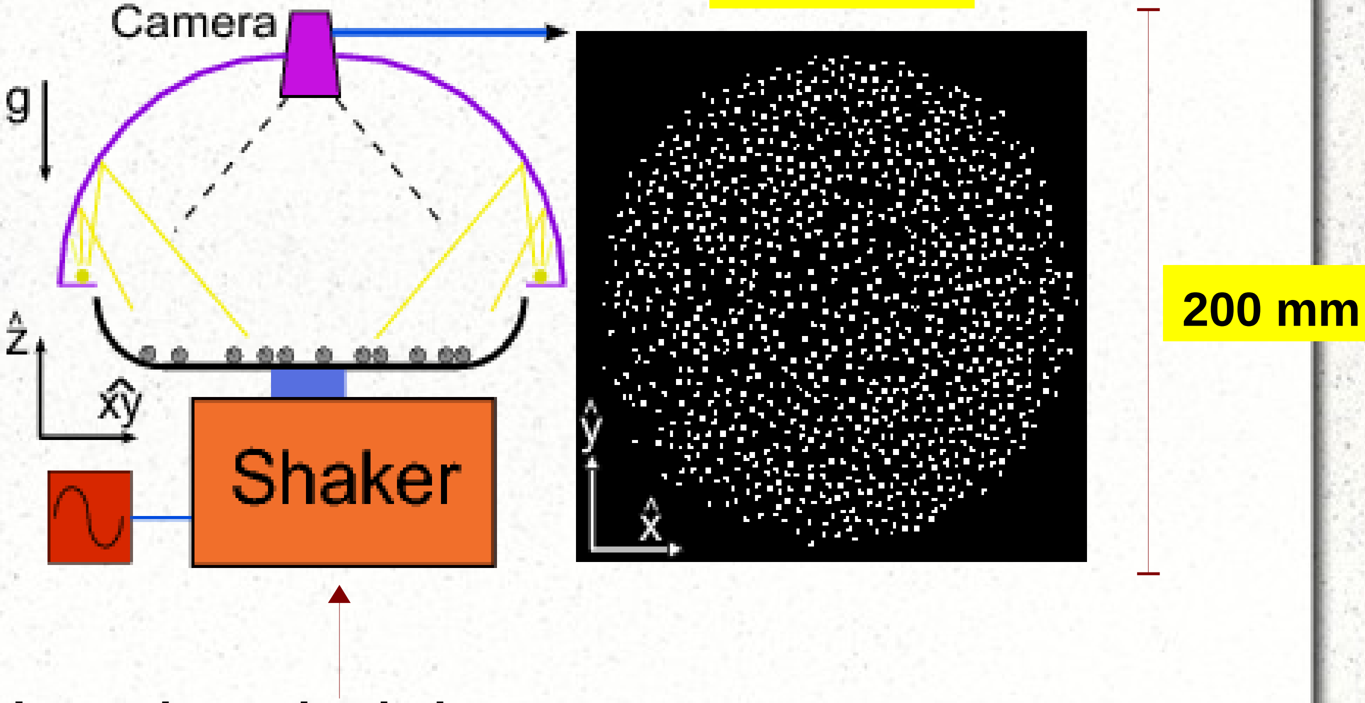
ENERGY LOSS :
inelastic collisions

Fast Camera
to observe xy motion of spheres

$\phi = 0.35$ Packing fraction

OUT OF EQUILIBRIUM

Rigid alluminium plate with a monolayer of steel **spheres** (diam 4mm)



Electrodynamic shaker:
Plate oscillations: sinusoidal signal z(t)=A sin(ωt),
A sphere stepping on another is a very rare event. **2D granular fluid.**

MODEL: INELASTIC HARD DISKS + RANDOM KICKS

Random kicks - Inelastic collisions (Van Noije *et al.*,'99)

$$m\dot{\mathbf{v}}_i = \underline{\xi_i} + \underline{\mathbf{F}_i^{coll}}$$

Random kicks $\langle \xi_i(t)\xi_j(t')\rangle = 2\,\Gamma\;\delta_{ij}\delta(t-t')$

Inelastic collisions $\mathbf{v}_i = \mathbf{v}_i' - \frac{(1+\alpha)}{2}\left[(\mathbf{v}_i' - \mathbf{v}_j')\cdot\hat{\boldsymbol{\sigma}}\right]\hat{\boldsymbol{\sigma}}$ $\longrightarrow$ $T_g = \frac{1}{N}\sum_{i=1}^{N}\langle \mathbf{v}_i^2\rangle$

Granular temperature

STATIONARY STATE $\boxed{T_g}$

$\alpha = 1 \rightarrow T_g = \infty$

Bad equilibrium limit

ONE FINITE ENERGY SCALE

MODEL: INELASTIC HARD DISKS + RANDOM KICKS + VISCOUS DRAG

Inelastic collisions - Random kicks – Viscous drag (Puglisi *et al.,'98*)

$$m\dot{\mathbf{v}}_i = \underline{-\gamma_b \mathbf{v}_i + \xi_i} + \underline{\mathbf{F}_i^{coll}}$$

Equilibrium thermostat

$$\langle \xi_i(t)\xi_j(t')\rangle = 2\ \gamma_b T_b\ \delta_{ij}\delta(t-t') \longrightarrow T_b$$

Thermostat temperature

Inelastic collisions

$$\mathbf{v}_i = \mathbf{v}_i' - \frac{(1+\alpha)}{2}\left[\left(\mathbf{v}_i' - \mathbf{v}_j'\right)\cdot\hat{\boldsymbol{\sigma}}\right]\hat{\boldsymbol{\sigma}} \longrightarrow T_g = \frac{1}{N}\sum_{i=1}^{N}\langle \mathbf{v}_i^2\rangle$$

Granular temperature

STATIONARY STATE

$$\boxed{T_g < T_b}$$

TWO FINITE ENERGY SCALES

$$\alpha = 1 \rightarrow T_g = T_b$$

Good equilibrium limit

COARSE-GRAINING OF DYNAMICS: HYDRODYNAMIC FIELDS

$$n(\mathbf{r},t)=\sum_i \delta_\epsilon(\mathbf{r}-\mathbf{r}_i(t)),$$

$$\mathbf{u}(\mathbf{r},t)=\frac{1}{n}\sum_i \mathbf{v}_i(t)\delta_\epsilon(\mathbf{r}-\mathbf{r}_i(t)),$$

$$T(\mathbf{r},t)=\frac{2m}{dn}\sum_i \frac{v_i^2(t)}{2}\delta_\epsilon(\mathbf{r}-\mathbf{r}_i(t)).$$

Fourier components of the fluctuations around the homogeneous stationary state

$$\delta\mathbf{a}(\mathbf{k},t)=\{\delta n(\mathbf{k},t),\delta T(\mathbf{k},t),u_{||}(\mathbf{k},t),u_\perp(\mathbf{k},t)\}$$

$$u_\perp(\mathbf{k})=\hat{k}_\perp\cdot\mathbf{u}(\mathbf{k})$$
$$u_{||}(\mathbf{k})=\hat{k}\cdot\mathbf{u}(\mathbf{k})$$

Longitudinal and transverse velocity field

Theory: Linear hydrodynamics equations + additive white noise

$$\delta\dot{\mathbf{a}}(\mathbf{k},t)=\mathbf{M}_{2\text{x}2}(\mathbf{k})\delta\mathbf{a}(\mathbf{k},t)+\underline{\xi}(\mathbf{k},t)$$

Noise correlators depend on the microscopic dynamics

$$\langle\xi_i(\mathbf{k},t)\xi_j^*(\mathbf{k}',t')\rangle=\delta(\mathbf{k}+\mathbf{k}')\delta(t-t')\delta_{ij}C_{ii}(\mathbf{k})$$

Structure of correlations in Fourier space

$$\langle\delta a_i(\mathbf{k})\delta a_j^*(\mathbf{k})\rangle$$

LINEARIZED HYDRODYNAMICS:
A LANGEVIN EQUATION FOR THE SHEAR MODES

SHEAR MODES ARE DECOUPLED FROM ALL OTHERS

Noise: FDT for each different source of dissipation

Internal noise: shear viscosity, granular temperature T_g

$$\langle \eta_{int}(\mathbf{k},t)\eta_{int}(\mathbf{k}',t')\rangle = 2\nu k^2 T_g \delta(t-t')\delta(\mathbf{k}+\mathbf{k}')$$

$$\dot{v}_\perp(\mathbf{k},t) = -(\gamma_b + \nu k^2)v_\perp(\mathbf{k},t) + \eta_{ext}(\mathbf{k},t) + \eta_{int}(\mathbf{k},t)$$

External noise: Thermostat temperature T_b

$$\langle \eta_{ext}(\mathbf{k},t)\eta_{ext}(\mathbf{k}',t')\rangle = 2\gamma_b T_b \delta(t-t')\delta(\mathbf{k}+\mathbf{k}')$$

LINEARIZED HYDRODYNAMICS:
A LANGEVIN EQUATION FOR THE SHEAR MODES

LINEAR LANGEVIN EQUATION FOR SHEAR MODES

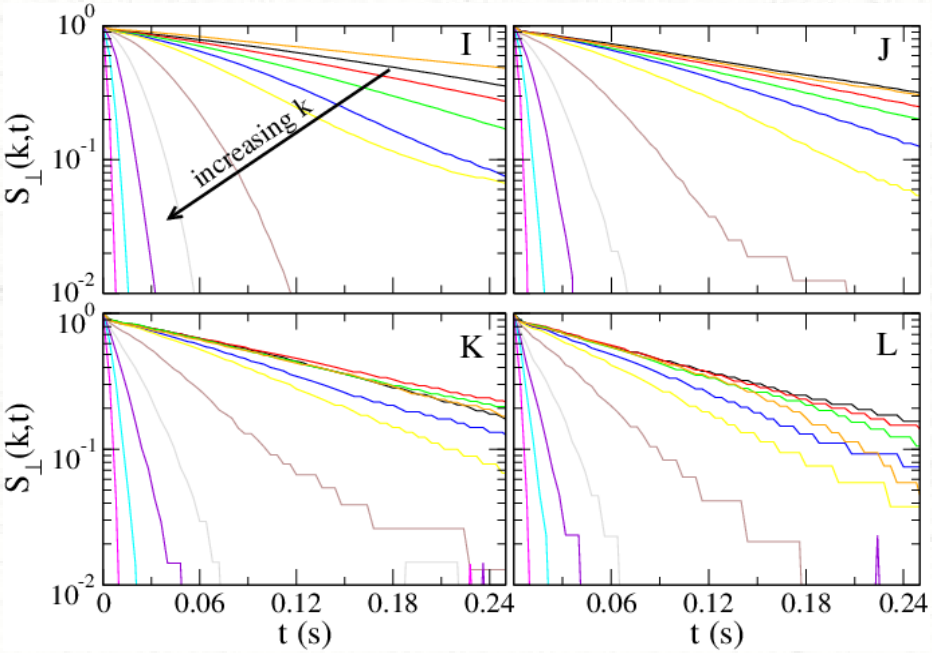
$$\langle u_\perp(\mathbf{k},t)u_\perp^*(\mathbf{k},0)\rangle \sim \; e^{-t/\tau(\mathbf{k})}$$

ϕ= 0.1

ϕ= 0.2

ϕ= 0.3

ϕ= 0.4



$$\dot{v}_\perp(\mathbf{k},t) = -(\gamma_b + \nu k^2)v_\perp(\mathbf{k},t) + \eta_{ext}(\mathbf{k},t) + \eta_{int}(\mathbf{k},t)$$

CORRELATION LENGTH IN THE VELOCITY FIELD

FOURIER SPECTRUM OF CORRELATIONS:

OBSERVABLE (SHEAR MODES) DEPENDENT EFFECTIVE TEMPERATURE

$$\boxed{nS_\perp(k) = N^{-1}\langle u_\perp(k)u_\perp(-k)\rangle = \frac{\gamma_b T_b + \nu k^2 T_g}{\gamma_b + \nu k^2} = T_g + \frac{(T_b - T_g)}{1+\xi^2 k^2} = T_{eff}(k)}$$

CORRELATIONS IN REAL SPACE (2D SYSTEM)

Two temperatures theoretical model

$$nG_\perp(\mathbf{r}) = T_g \delta^{(2)}(\mathbf{r}) + \underline{(T_b - T_g)}\frac{K_0(r/\xi)}{\xi^2}$$

$$\boxed{\xi = \sqrt{\nu/\gamma_b}}$$

$$K_0(r/\xi) \to \sqrt{\frac{\pi}{2}}\frac{e^{-r/\xi}}{(r/\xi)^{1/2}}$$

"Distance" from equilibrium: **AMPLITUDE** of correlations

Shear viscosity: **RANGE** of correlations

GG, A.Sarracino,D.Villamaina, A.Puglisi, *EPL,* 96, (2011)

GG, A.Sarracino, D.Villamaina, A.Puglisi, *J. Stat. Mech.*,P08017 (2011)

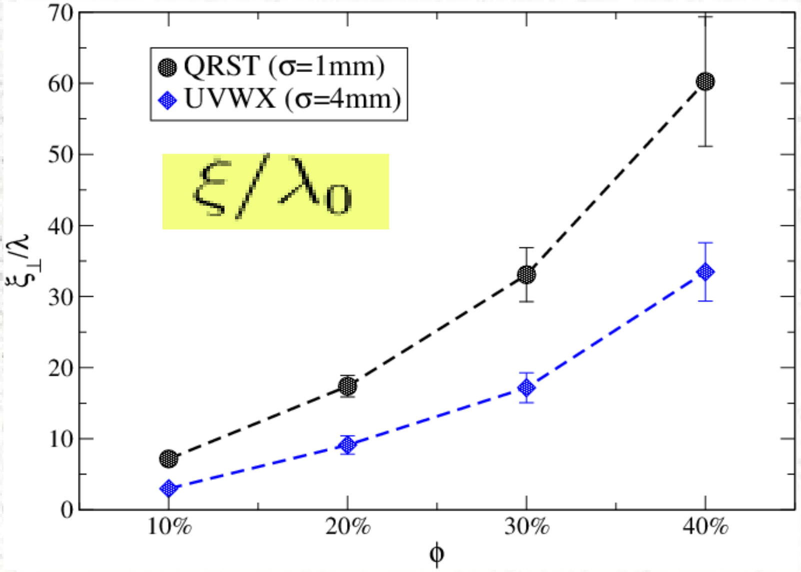
THEORY AND EXPERIMENTS

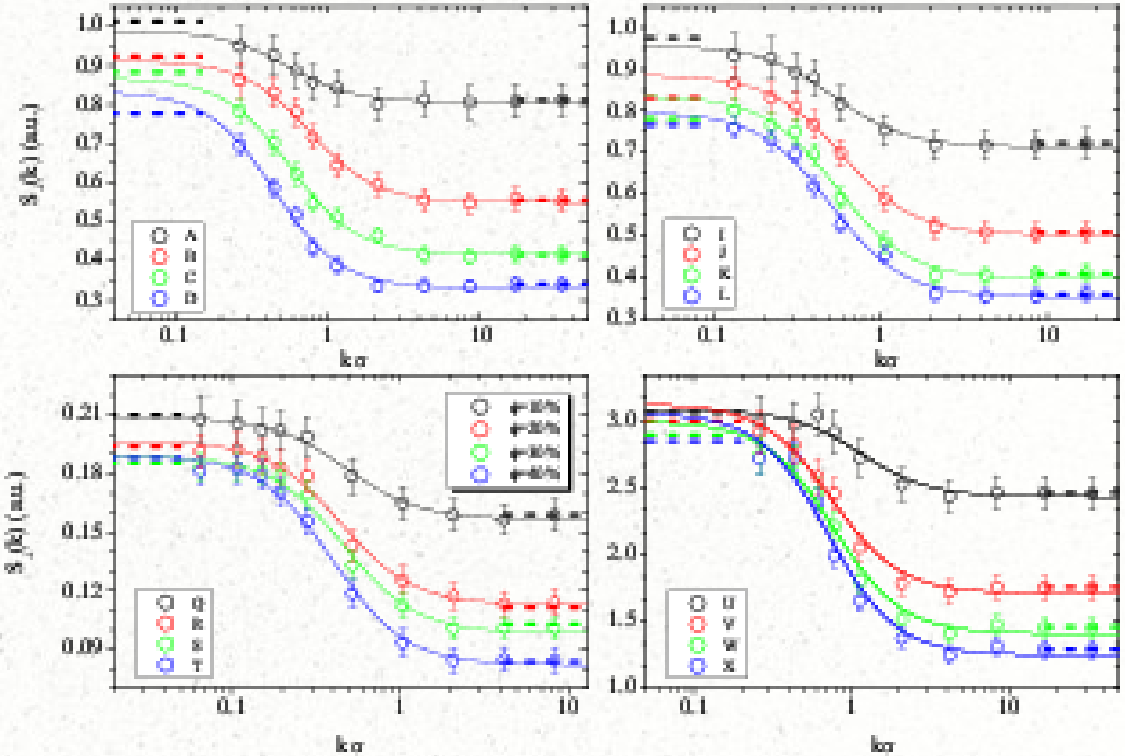
A.Puglisi, A.Gnoli, GG, A.Sarracino, D.Villamaina, *J. Chem. Phys.* 136, (2012)

Driving with random kicks and viscous drag

$$N^{-1}\langle u_\perp(\mathbf{k})u_\perp^*(\mathbf{k})\rangle = T_g + \frac{T_b - T_g}{1 + k^2\xi^2}$$

$$\boxed{\xi = \sqrt{\nu/\gamma_b}}$$





Driving only with random kicks

$$N^{-1}\langle u_\perp(\mathbf{k})u_\perp^*(\mathbf{k})\rangle \sim 1/k^2$$

T. van Noije *et al*, Phys. Rev. E 59, (1999)

OUT-OF-EQUILIBRIUM THE **RANGE** OF CORRELATIONS GROWS WITH THE **PACKING FRACTION** ϕ

Not compatible with our data

ENTROPY PRODUCTION IN THE STATIONARY STATE

Linear system of coupled Langevin equations

$$\delta\dot{\mathbf{a}}(\mathbf{k},t) = \mathbf{M}_{2\mathrm{x}2}(\mathbf{k})\delta\mathbf{a}(\mathbf{k},t) + \underline{\xi}(\mathbf{k},t)$$

Trajectory in space of hydrodynamic variables

$$\Omega_0^t = \{\delta\mathbf{a}(\mathbf{k},s)\}_0^t$$

$$P(\Omega_0^t) = \exp\left[-\int_0^t ds \sum_i \frac{1}{C_{ii}(\mathbf{k})}|\delta\dot{a}_i(\mathbf{k},s) - M_{ij}(\mathbf{k})\delta a_j(\mathbf{k},s)|^2\right]$$

Onsager-Machlup formula for trajectories probability

ENTROPY PRODUCTION

$$\frac{1}{t}\log\left[\frac{P_{\mathbf{k}}(\Omega_0^t)}{P_{\mathbf{k}}(\overline{\Omega}_0^t)}\right] \sim \frac{1}{t}\int_0^t d\tau\ \Delta s_m(\mathbf{k},\tau)$$

Backward trajectory

$$\overline{\Omega}_0^t = \{\epsilon\delta\mathbf{a}(\mathbf{k},t-s)\}_0^t$$

A.Puglisi, D.Villamaina
EPL, 88 (2009)

ENTROPY PRODUCTION AND MACROSCOPIC OBSERVABLES

$\boxed{\text{ENTROPY PRODUCTION}}$

$$\frac{1}{t}\log\left[\frac{P_{\mathbf{k}}(\Omega_0^t)}{P_{\mathbf{k}}(\overline{\Omega_0^t})}\right] \sim \frac{1}{t}\int_0^t d\tau\ \Delta s_m(\mathbf{k},\tau)$$

$$\boxed{\Delta s_m(\mathbf{k},\tau) = F_{\mathbf{k}}^{diss}(\alpha,\mathbf{\Gamma},\mathbf{D})\ \mathcal{R}[\delta\rho_{\mathbf{k}}(\tau)\dot{T}_{-\mathbf{k}}(\tau)]}$$

α Restitution coefficient

$\mathbf{\Gamma}$ Set of transport coefficients

$\mathbf{D}$ Set of thermostat parameters

Constant term
"Driving force"

$$F_{\mathbf{k}}^{diss}(\alpha,\mathbf{\Gamma},\mathbf{D}) \qquad F_{\mathbf{k}}^{diss}(\alpha=1,\mathbf{\Gamma},\mathbf{D})=0$$

EQUILIBRIUM

Fluctuating term
non-equilibrium "current"

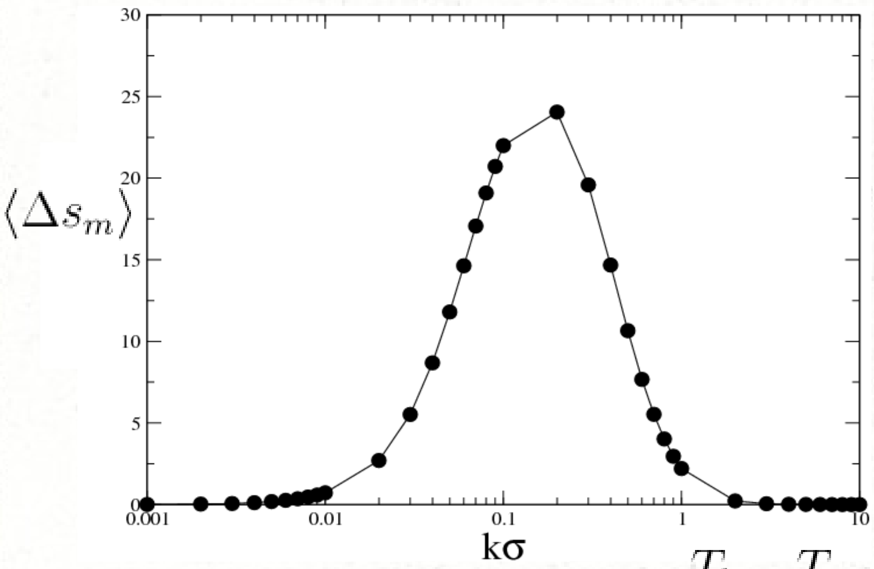
$$\mathcal{R}[\delta\rho_{\mathbf{k}}(\tau)\dot{T}_{-\mathbf{k}}(\tau)] \qquad \langle\mathcal{R}[\delta\rho_{\mathbf{k}}(\tau)\dot{T}_{-\mathbf{k}}(\tau)]\rangle = 0$$

ENTROPY PRODUCTION FROM LINEAR HYDRODYNAMICS

$$\boxed{\frac{1}{t}\int_0^t d\tau\ \Delta s_m(\mathbf{k},\tau) \to F_{\mathbf{k}}^{diss}\ \langle \mathcal{R}[\delta\rho_{\mathbf{k}}\delta\dot{T}_{-\mathbf{k}}]\rangle}$$

G. Gradenigo, A. Puglisi, A.Sarracino, *arXiv:1205.3639*

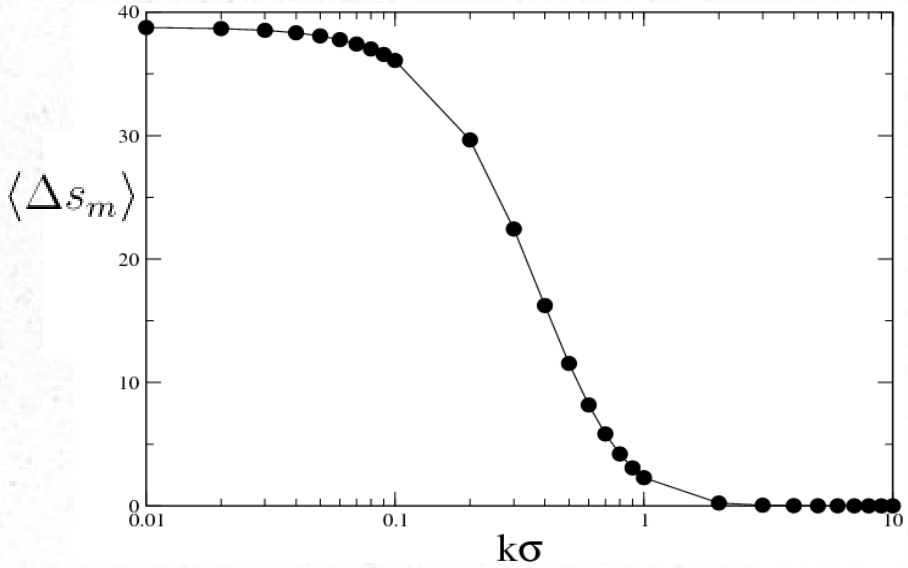
Random kicks + viscous drag



$$N^{-1}\langle u_\perp(\mathbf{k})u_\perp^*(\mathbf{k})\rangle = T_g + \frac{T_b - T_g}{1+k^2\xi^2}$$

Finite range off-equilibrium correlations

Only random kicks



$$N^{-1}\langle u_\perp(\mathbf{k})u_\perp^*(\mathbf{k})\rangle \sim 1/k^2$$

Scale free off-equilibrium correlations

CONCLUSIONS

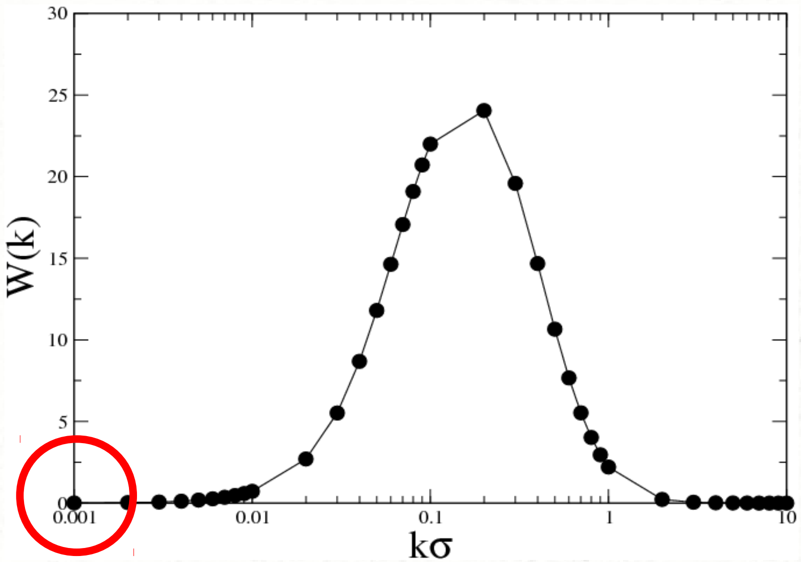
- Non-equilibrium correlations for hydrodynamic fields in a driven granular fluid

- Observable dependent effective temperature

- Stochastic bath with friction: agreement with experimental results

- Relation between stationary entropy production and out-of-equilibrium correlations.

- Entropy production can be calculated for every system described by a set of coupled linear langevin equations: flocking birds, swarms, swimming bacteria … active matter !



LARGE SCALE BEHAVIOUR OF ENTROPY PRODUCTION

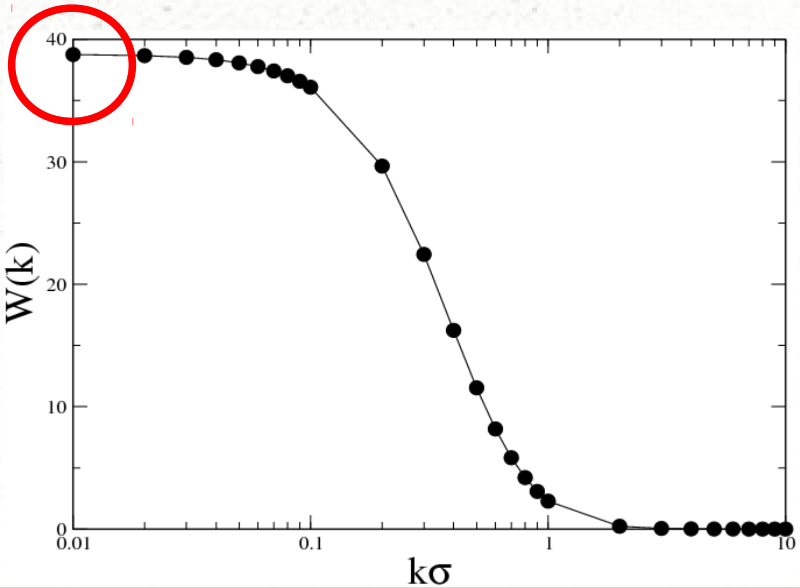
$$\langle \rho_{\mathbf{k}}(s)\dot{T}_{-k}(s)\rangle \sim k^2$$

$$F^{diss}_{\mathbf{k}}(\alpha) = a + b\ k^2 + \mathcal{O}(k^4)$$

$$\boxed{\langle W_{\mathbf{k}}(s)\rangle = F^{diss}_{\mathbf{k}}(\alpha)\ \langle \delta\rho_{\mathbf{k}}(s)\dot{T}_{-k}(s)\rangle}$$

$$\langle \rho_{\mathbf{k}}(s)\dot{T}_{-k}(s)\rangle = C + \mathcal{O}(k^2)$$

$$F^{diss}_{\mathbf{k}}(\alpha) = a_1 + b_1\ k^2 + \mathcal{O}(k^4)$$

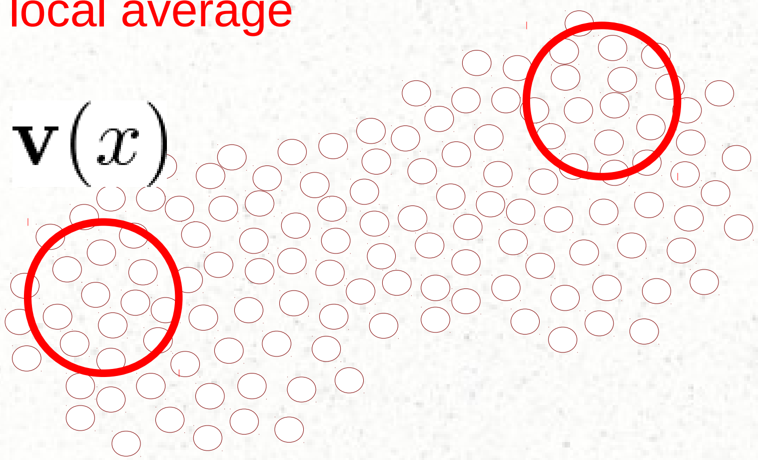


LARGE SCALE CORRELATIONS: STUDY OF HYDRODYNAMIC FIELDS

Define "fields" from a local average

$\mathbf{v}(y)$

$\mathbf{v}(x)$



$$\delta\mathbf{v}(x) = \mathbf{v}(x) - \langle\mathbf{v}(x)\rangle$$

In equilibrium fluid

$$\langle\mathbf{v}(x)\mathbf{v}(y)\rangle = \delta(x-y)$$

Out of equilibrium

$$\langle\mathbf{v}(x)\mathbf{v}(y)\rangle = \ ?$$

In Fourier space things are simpler …

Transverse velocity modes

$$v_\perp(\mathbf{k}) = \sum_i^N e^{i\mathbf{k}\cdot\mathbf{r}_i}(\mathbf{v}_i \cdot \hat{\mathbf{k}}_\perp) \qquad \hat{\mathbf{k}}\cdot\hat{\mathbf{k}}_\perp = 0$$

COARSE-GRAININIG OF DYNAMICS: HYDRODYNAMIC FIELDS

$$\mathbf{a}(k,t) = \begin{pmatrix} \delta\varrho(k,t) \\ \delta T(k,t) \\ \delta v_{||}(k,t) \\ \delta v_{\perp}(k,t) \end{pmatrix}$$

Fluctuations around homogeous stationary state

Linearized fluctuating hydrodynamics

$$\dot{\mathbf{a}}(k,t) = \mathbf{M}(k)\mathbf{a}(k,t) + \underline{\boldsymbol{\Theta}(k,t)}$$

White noise
Fields not coupled by noise

DYNAMICAL MATRIX

$$\mathbf{M}(k) = - \begin{array}{c} \begin{array}{cccc} \rho & T & v_{||} & v_{\perp} \end{array} \\ \begin{pmatrix} 0 & 0 & \imath kn & 0 \\ \gamma_0\omega_c g(n)T_g/n & 3\gamma_0\omega_c + D_T k^2 + 2\gamma_b/m & \imath 2kp/dn & 0 \\ \imath k v_T^2/n & \imath kp/\rho T_g & \nu_l k^2 + \gamma_b/m & 0 \\ 0 & 0 & 0 & \nu k^2 + \gamma_b/m \end{pmatrix} \end{array} \begin{array}{c} \rho \\ T \\ v_{||} \\ v_{\perp} \end{array}$$

LARGE SCALE CORRELATIONS: STUDY OF HYDRODYNAMIC FIELDS

$$\mathbf{a}(k,t) = \begin{pmatrix} \delta\varrho(k,t) \\ \delta T(k,t) \\ \delta v_{||}(k,t) \\ \delta v_{\perp}(k,t) \end{pmatrix}$$

Linearized fluctuating hydrodynamics

$$\dot{\mathbf{a}}(k,t) = \mathbf{M}(k)\mathbf{a}(k,t) + \underline{\boldsymbol{\Theta}(k,t)}$$

Fluctuations around homogeous stationary state

White noise
Fields not coupled by noise

DYNAMICAL MATRIX

Granular dissipation

$$\mathbf{M}(k) = - \begin{array}{cc} \begin{array}{cccc} \rho & T & v_{||} & v_{\perp} \end{array} & \\ \begin{pmatrix} 0 & 0 & \imath kn & 0 \\ \gamma_0\omega_c g(n)T_g/n & 3\gamma_0\omega_c + D_T k^2 + 2\gamma_b/m & \imath 2kp/dn & 0 \\ \imath k v_T^2/n & \imath kp/\rho T_g & \nu_l k^2 + \gamma_b/m & 0 \\ 0 & 0 & 0 & \nu k^2 + \gamma_b/m \end{pmatrix} & \begin{array}{c} \rho \\ T \\ v_{||} \\ v_{\perp} \end{array} \end{array}$$

DEGREE OF EQUIPARTITION

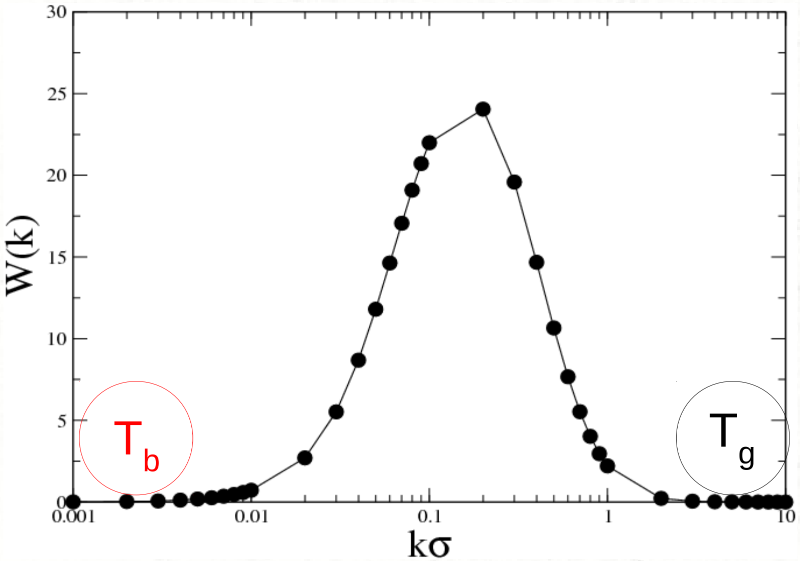
$$\boxed{nS_\perp(k) = \langle v_\perp(\mathbf{k}) v_\perp^*(\mathbf{k}) \rangle = \sum_{i,j} \langle e^{i\mathbf{k}(\mathbf{r}_i - \mathbf{r}_j)} (\mathbf{v}_i \cdot \hat{k}_\perp)(\mathbf{v}_j \cdot \hat{k}_\perp) \rangle = T_{eff}(k)}$$

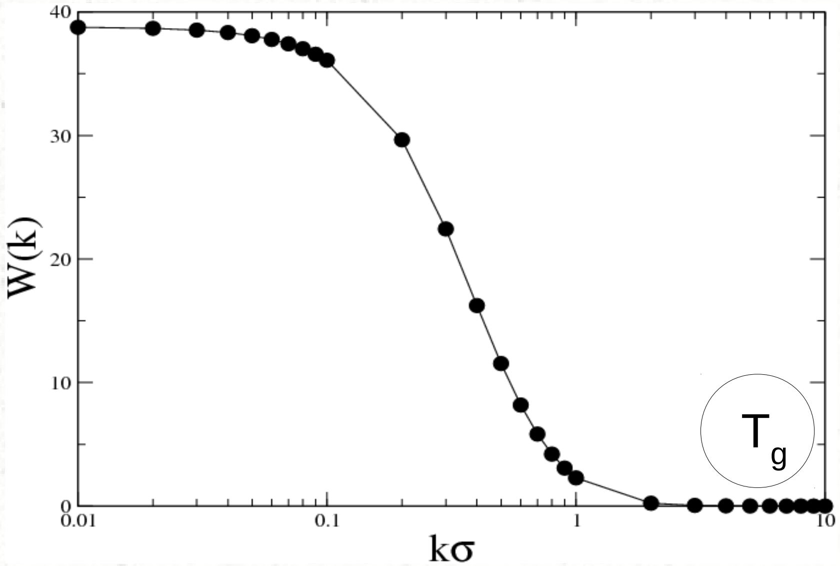
	small *k*	large *k*
Driving with random kicks and viscous drag	$T_{eff}(k \to 0) = T_b$ Each mode has a **<u>the same</u>** typical energy	T_g
Driving with ***only*** random kicks	$T_{eff}(k \to 0) \sim \frac{\Gamma}{k^2}$ Each mode has a **<u>different</u>** typical energy	T_g

ENTROPY PRODUCTION FROM LINEAR HYDRODYNAMICS

$$T_{eff}(k \to 0) = T_b$$

$$T_{eff}(k \to 0) \sim \frac{\Gamma}{k^2}$$





Low k modes : all with typical energy T_b

High k modes : all with typical energy T_g

Low k modes : each one with a ***different typical energy***

High k modes : all with typical energy T_g

OUTLINE OF THE TALK

- Experimental setup for a driven granular fluid
- How to model the dynamics ? Two microscopic energy injection mechanisms
- Coarse grained study of the dynamics: linear fluctuating hydrodynamics
- Static correlations of hydrodynamic fields (comparison with experiments): a landmark of non-equilibrium
- Entropy production for hydrodynamic fields
- Conclusions